

RELATIVE IMPORTANCE ANALYSIS WITH MULTIVARIATE MODELS: SHIFTING THE FOCUS FROM INDEPENDENT VARIABLES TO PARAMETER ESTIMATES

Joseph N. Luchman*¹, Xue Lei² and Seth Kaplan³

¹Fors Marsh Group, 1010 N. Glebe Road, Suite 510, Arlington, Virginia

^{2,3}George Mason University, Department of Psychology, 4400 University Drive, Fairfax, Virginia

*jluchman@forsmarshgroup.com

ABSTRACT

Conclusions regarding the relative importance of different independent variables in a statistical model have meaningful implications for theory and practice. However, methods for determining relative importance have yet to extend beyond statistical models with a single dependent variable and a limited set of multivariate models. To accommodate multivariate models, the current work proposes shifting away from the concept of independent variable relative importance toward that of parameter estimate relative importance (PERI). This paper illustrates the PERI approach by comparing it to the evaluation of regression slopes and independent variable relative importance (IVRI) statistics to show the interpretive and methodological advantages of the new concept and associated methods. PERI's advantages above standardized slopes stem from the same fit metric that is used to compute PERI statistics; this makes them more comparable to one another than standardized slopes. PERI's advantages over IVRI stem from situations where independent variables do not predict all dependent variables; hence, PERI permits importance determination in situations where independent variables are nested in dependent variables they predict. We also provide recommendations for implementing PERI using dominance analysis with statistical models that can be estimated with maximum likelihood estimation combined with a series of model constraints using two examples.

Keywords: *Relative Importance, Dominance Analysis, Path Analysis, Zero-inflated Poisson Regression, Multivariate Model*

INTRODUCTION

Behavioral science theory grows more complex every year and statistical techniques to test such theory continue to grow similarly complex. Consider the common practice of incorporating multiple independent and dependent variables in a single statistical model such as a structural equation or path model (e.g., Muthén, 1984). In such multivariate models, researchers use regression slope parameter estimates as confirmatory evidence for the conceptual relationships among the multiple variables as described by an underlying theory.

In addition to providing evidence of relationships, researchers also use statistical models to assess the importance of different variables in their conceptual model. Evaluating importance often involves determining how variables contribute to model fit. However, evaluating relative contributions to model fit when independent, or dependent, variables are correlated is not straightforward (see Budescu, 1993; Grömping, 2007). In these cases, researchers can use specialized relative importance analysis methods (Johnson & LeBreton, 2004). Relative importance analysis methods parse independent variables' contributions to fit even when variables are correlated (e.g., LeBreton & Tonidandel, 2008).

Specialized relative importance analysis methods have been applied mostly to models with one dependent variable (e.g., Azen & Traxel, 2009; Tonidandel & LeBreton, 2010) and extended to limited types of multivariate models (e.g., multivariate linear regression; Azen & Budescu, 2006; LeBreton & Tonidandel, 2008). As we note above, multivariate models are increasingly common, and we seek to expand relative importance analysis to more types of multivariate models. Specifically, in this work we extend relative importance analysis to models with multiple independent and dependent variables irrespective of the distribution of dependent variables; that is, this method extends to non-Gaussian/non-normally distributed variables (e.g., generalized path models; Rabe-Hesketh, Skrondal, & Pickles, 2004).

In the process of extending relative importance to other multivariate models, we also broaden the concept of relative importance. We suggest an alternative and more expansive conceptualization of relative importance, parameter estimate relative importance (PERI). We argue that PERI is more appropriate for determining importance in many multivariate models such as structural equation or path models than the prevailing independent variable relative importance (IVRI) approach.

In addition, we illustrate how to implement PERI using dominance analysis (DA; Budescu, 1993) through two data analytic examples. In these illustrations we also compare PERI with standardized and other transformed parameter estimates to distinguish how PERI-based DA statistics differ from standardized and transformed parameter estimates in model interpretation.

The remainder of the paper unfolds as follows: We begin by briefly reviewing relative importance analysis focusing on the methods that extend relative importance analysis to multivariate models. Following that, we introduce PERI, a reconceptualization and extension of IVRI. In differentiating PERI from IVRI, we highlight how PERI can overcome key limitations of IVRI for multivariate models. We also describe the primary method for adapting DA for use in computing PERI statistics by constraining parameter estimate values to zero during estimation. We then walk through two data analytic examples of PERI using the DA methodology. As a part of the discussion of each example, we compare standardized parameter estimates, as well as IVRI statistics for each model, to PERI statistics to highlight the role of each more clearly. Finally, in the Discussion section, we note other potential applications of PERI, discuss practical issues in implementing PERI, as well as point to areas where there is a need for future research.

RELATIVE IMPORTANCE ANALYSIS FOR LINEAR REGRESSIONS

Relative importance is defined as “the proportionate contribution each [independent variable] makes to R^2 , considering both its direct effect (i.e., its correlation with the [dependent variable]) and its effect when combined with the other [independent] variables in the regression equation” (Johnson & LeBreton, 2004 p. 240). This definition of relative importance, as well as most methods to determine relative importance of independent variables, developed from applications to linear regression (Genizi, 1993; Grömping, 2007; Johnson & LeBreton, 2004).

An important point to note is that linear regression is a single equation model. We depict what is meant by a single equation model, in terms of its model structure, in Figure 1, Model 1a (cf.

LeBreton & Tonidandel, 2008; Figure 1). As seen in Model 1a, single equation models like linear regression have one dependent variable and one regression slope parameter estimate per independent variable, represented by each path in the figure.

As is noted above, one method recommended by methodologists to determine IVRI for linear regression is DA (Budescu, 1993). DA quantifies and ranks the importance of all the independent variables in a linear regression based on the amount of the R^2 that is associated with each regression slope. The DA method involves obtaining the R^2 values for all possible combinations of independent variables either estimated in the model (i.e., the independent variable has an estimated slope parameter) or omitted from the model (i.e., the independent variable does not have an estimated slope parameter). Consequently, the DA method is similar to a factorial experiment; the independent variables are included one, and then two, then three at a time and so on until all combinations of independent variables are included in the model. In obtaining the full-factorial of combinations across all independent variables, DA can separate their contributions to model fit even when independent variables are correlated.

Before moving on we note that because most relative importance methods, such as DA, were developed from linear regression, the idea that a single independent variable implies a single slope parameter estimate is built into the current definition of relative importance. In the coming sections, we outline how single equation relative importance methods have been extended to statistical models for multiple dependent variables. Below, we begin by discussing how these multivariate models retain an independent variable focus despite having multiple parameter estimates per independent variable. Additionally, we note shortcomings of retaining the independent variable focus when extended to other multivariate models.

RELATIVE IMPORTANCE ANALYSIS FOR MULTIVARIATE MODELS

Multivariate Relative Importance Methods: Current State of the Art

Relative importance analysis has been extended to a limited set of multivariate models. A constraint of such extensions from linear regression is that the structure of the multivariate model must permit the isolation of each independent variable on the entire set of dependent variables; thus, such models must permit independent variables to estimate a regression slope that predicts each dependent variable. A graphical depiction of the multivariate models to which relative importance has been extended is displayed in Figure 1; Model 1b.

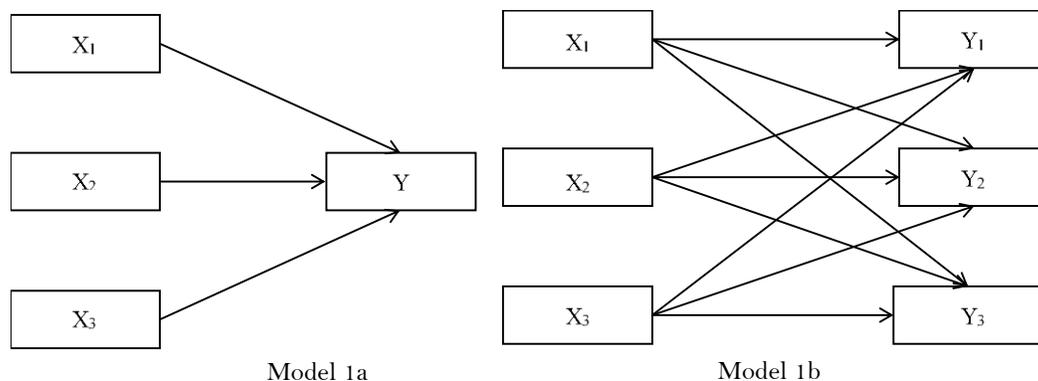


Figure 1. Models for Independent Variable-based Relative Importance

Model 1b matches the structure of multivariate linear regression to which DA has been extended (Azen & Budescu, 2006). In multivariate linear DA, the importance of an independent variable in terms of model fit is determined across the set of dependent variables in a way similar to that of single equation linear regression. To be specific, in multivariate linear DA all of the regression slope parameters associated with a single independent variable are either estimated simultaneously or omitted from the model simultaneously; thus the parameter estimates associated with an independent variable are treated as a set. Hence, the full-factorial experiment-like structure of DA is generalized from linear regression where a single slope parameter is associated with an independent variable to multivariate linear regression where multiple slope parameters are associated with the same independent variable. Generalizing DA from a single parameter to a set of slope parameter estimates has also been applied to multinomial logistic regression (Luchman, 2014); an extension of logistic regression that has $k-1$ prediction equations where k is the number of categories in the dependent variable.

Limitations of Independent Variable-based Multivariate Relative Importance Methods

Many real theories and research questions have a structure that cannot or do not fit with those outlined in Figure 1. That is, many theoretical models applied to real theories do not require, conceptually, that every independent variable predict every dependent variable. To illustrate our point, consider, for instance, the models depicted in Figure 2. Model 2a has a mediated structure found in many theoretical models and yet is structurally similar to the Model 1a. In fact, both have the same number of regression slope parameter estimates. The only difference between the two models is that in Model 1a, X_1 predicts Y whereas in Model 2a, X_1 predicts X_2 .

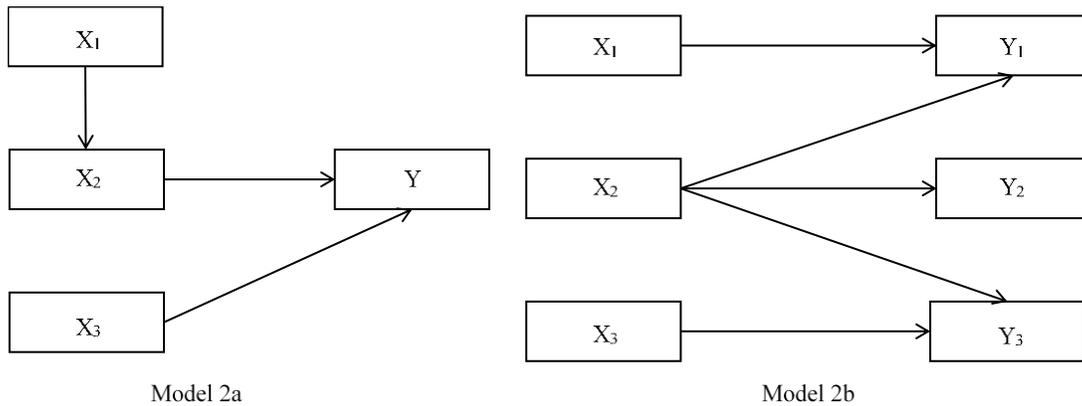


Figure 2. Models where Independent Variables are Nested in Dependent Variables they Predict

Although similar in structure, Models 1a and 2a have different implications with respect to determining IVRI. In particular, Model 1a is a standard linear regression in which all independent variables are related to a single dependent variable. In this case, all the independent variable comparisons (i.e., X_1, X_2, X_3) are contrasted against one another in terms of how they explain variance in the single dependent variable Y . In Model 2a, the comparisons change, as each independent variable no longer predicts the same dependent variable. Specifically, X_1 predicts X_2 , whereas X_2 and X_3 predict Y . Consequently, Model 2a will result in comparing independent variables nested in the dependent variable which they predict. This is because the source of variance explained by X_1 in Model 2a differs from the source of variance explained by X_2 and X_3 .

The issue of how to determine IVRI in a Model 2a is not clear from the extant literature. We argue however that the potential for independent and dependent variables being nested in one another in multivariate statistical models is a distinct limitation of adapting the extant IVRI concept to determining relative importance for multivariate models. To be more specific, in cases where independent variables predict different dependent variables, the researcher cannot disentangle the predictive usefulness of the independent variable from characteristics of the dependent variable being predicted. By contrast, in Model 1a, IVRI is attributable to independent variables only as they all predict the same dependent variable.

Consider now the Model 2b. Model 2b is similar to Model 1b except that independent variable X_1 does not predict Y_2 or Y_3 . Similarly, independent variable X_3 does not predict Y_1 or Y_2 . In other words, Model 2b is a more parsimonious model than the Model 1b as it estimates fewer parameters. Similar to Model 2a, Model 2b depicts a model where all the independent variables do not predict all the same dependent variables. Moreover, Model 2b also does not require that independent variables predict the same number of dependent variables as X_2 predicts all three whereas both X_1 and X_3 predict only one dependent variable. In fact, for Model 2b, an IRVI statistic would combine the predictive usefulness of the independent variable, the number of dependent variables predicted by the independent variables, and characteristics of the dependent variables being predicted (i.e., distributional properties like variance). By comparison, IVRI determination for Model 1b is conceptually more reasonable because each independent variable predicts each dependent variable. There is no nesting of independent variable and dependent variable relationships in Model 1b; all independent variables have the opportunity to explain variance in each dependent variable.

INTRODUCING PARAMETER ESTIMATE RELATIVE IMPORTANCE

We address the limitations outlined above in the IVRI approach by developing a different conceptualization of relative importance. In particular, we argue that conceptual problems such as nesting of independent and dependent variables in attempting to apply IVRI to multivariate models can be remedied by a conceptual shift toward considering the relative importance of parameter estimates rather than the relative importance of independent variables.

We propose the following definition of PERI for use in behavioral science. PERI is, “the determination that a parameter estimate contributes more to model fit than another parameter estimate, as aggregated across nested models within the same statistical model.” There are several components of this definition that warrant elaboration, especially in distinguishing this approach from IVRI analysis.

First, the focus of PERI is on model fit as ascribed to parameter estimates, not to independent variables. This focus on specific parameter estimates avoids the possible nesting of independent and dependent variables discussed above for models in which all independent variables do not predict all dependent variables. Furthermore, the focus on parameter estimates permits the incorporation of estimates which do not have a distinction between independent and dependent variable, such as a covariance.

Second, the focus of PERI is on individual parameter estimates. This is important for situations such as those in Models 1b and 2b where an IVRI approach explicitly groups together collections of parameter estimates to reflect the relative importance of the independent variable associated with each on the set of dependent variables. By contrast, the PERI definition does not require the grouping of parameter estimates. Instead, each parameter estimate associated with each independent – dependent variable pair can result in its own importance statistic. More centrally, the PERI approach offers researchers more information than the IVRI approach when parameters

are grouped by showing how different independent – dependent variable pairs contribute to explaining model fit.

Third, PERI relaxes the necessity to explicitly use an additive decomposition of the R^2 metric. Rather than require the use of an R^2 metric, we recommend using the more general term of model fit metric. The generalization to model fit metrics broadly permits the use of useful fit metrics that are not scaled as proportions (e.g., complete dominance, a binary or yes/no indicator). Additionally, we believe that by broadening the use of fit metrics, we avoid requiring importance statistics to be proportionate; that is, that the importance statistics will be parts of a whole. Proportionate fit applies best to specific importance statistics such as general dominance statistics (discussed below) which are additive decompositions of the model fit metric. However, we believe that additional statistics computed by DA can be considered to be valid PERI metrics as well. For example, binary statistics such as complete dominance or other non-proportion-based metrics such as conditional dominance statistics (Budescu, 1993; Budescu & Azen, 2004) which are not, strictly speaking, proportions of the R^2 are permitted to be valid PERI statistics.

Finally, PERI dictates that comparisons between parameter estimates are “aggregated across nested models within the same statistical model.” This language was used to suggest that the model fit metrics will be incorporated using PERI in a way similar to that outlined in the IVRI approach. Johnson and LeBreton (2004) noted that direct and indirect effects, combined with other independent variables in the model, should be used to determine variable importance. Our choice to use the term “aggregated across nested models within the same statistical model” is intended to be more general and to imply the same kind of relations - aggregating model fit metrics across sub-models nested within the full model to make the final determination of importance.

Having defined PERI, we transition in the following section to outlining the recommended DA method researchers can use to determine PERI in multivariate models. Following the discussion of DA, this section concludes with two data analytic examples outlining PERI as computed using DA in detail.

PARAMETER ESTIMATE RELATIVE IMPORTANCE WITH DOMINANCE ANALYSIS

Recommended Method for Determining PERI

We recommend the use of DA to determine PERI. DA is a well-known methodology among behavioral scientists and is recommended as a relative importance analysis approach in several authoritative articles (e.g., Grömping, 2007; Johnson & LeBreton, 2004). DA is also a flexible method in terms of the statistical model that can be applied and the way in which the model fit metrics are aggregated to produce dominance statistics.

Although we recommend DA for PERI determination, we acknowledge that standardized parameter estimates could be used to compare parameter estimates in multivariate models. Standardized parameter estimates are regression slope statistics that have been re-scaled to be comparable to one another within a statistical model, often by scaling the estimate to be associated with a standard deviation change on the independent variables. Comparing the magnitude of standardized parameter estimates to infer importance of the respective independent variables or parameters also has a long history in behavioral science (see Greenland, Maclure, Schlesselman, Poole, & Morgenstern, 1991 for a discussion and criticism of their use).

The primary drawback of examining standardized parameter estimates for determining importance is that these parameters are intended to describe predicted values of the dependent variable given changes in the independent variable, but not to describe model fit as such.

Standardized parameter estimates, as re-scaled slope coefficients, provide the location of predicted dependent variable values along the continuum of the independent variable and thus consider each independent variable-dependent variable relationship in the context of one standard deviation of change. By contrast, PERI-based DA describes the impact of the independent-dependent variable relationship across the entire continuum of the independent variable involved in the parameter estimate as it is based on a model fit metric.

Standardized parameter estimates are also complicated to compare across multiple dependent variables. This is because the predicted values described by standardized parameter estimates only offer information about the location of dependent variable being predicted. This can be compared with PERI-based DA which describes how much information is contributed to understanding the entire model owing to the parameter estimate; again, this is because PERI-based DA is based on a model fit metric.

We thus believe PERI-based DA statistics are a conceptually more useful and informative set of metrics for determining importance in multivariate models than standardized parameter estimates. We revisit the use of standardized parameter estimates below and compare them to DA statistics to in the context of our two data analytic examples.

DA Extended to Multivariate Models: A Constrained Maximum Likelihood Approach

DA is an ensemble method or an approach that combines a set of metrics estimated from multiple statistical models into a single statistic (e.g., Zhou, 2012). For IVRI, the ensemble set includes model fit metrics for all possible combinations of independent variables either estimated from the data or omitted from the model. A statistical model with p independent variables will then include model fit metrics for all $2^p - 1$ models—which represent all combinations of estimated or omitted status (hence, base 2) for all p independent variables removing the model in which no independent variables are included (hence, minus 1).

Extending DA to PERI is straightforward and involves either estimating a parameter from the data or constraining the estimate to 0; this latter condition being tantamount to omitting the parameter estimate as would be done with an independent variable in IVRI. To omit a specific parameter, estimate from a model, the researcher must use an estimator which permits estimation constraints. Maximum likelihood is one such estimator (see Gould, Pitbaldo, & Poi, 2010). Maximum likelihood estimation is a common estimator, is used across many statistical analyses in behavioral science and is the estimator we recommend for use in DA to determine PERI.

To omit a parameter from the model, the parameter estimate must be constrained to have a value of 0 as this will effectively omit it and it will not contribute to improving model fit. The DA procedure to estimate PERI then requires obtaining fit metrics across all possible combinations of 0 constraints applied to parameter estimates in for the focal statistical model. This parallels the IVRI approach in which all possible combinations of independent variables would need to be omitted. For PERI determination, a statistical model with d parameter estimates will then have to estimate model fit metrics associated with all $2^d - 1$ models. Again, this will represent all combinations of estimated or constrained-to-0 status for all d parameters (less the no parameter estimates estimated model).

After obtaining the full-factorial of model fit metrics estimating or omitting each parameter estimate, the researcher can compute dominance statistics. The *general dominance statistic* computed from the ensemble of fit metrics is the most commonly reported DA statistic and is the

focus in the present work. The general dominance statistic is, of the three dominance statistics that can be computed (see Budescu, 1993; Budescu & Azen, 2004), the easiest to interpret because it represents the averaged contribution to model fit by a parameter estimate. An additional useful property of the general dominance statistics is that they sum to the total fit metric across all parameter estimates. Thus, the general dominance statistic associated with each parameter estimate is the component of the total fit metric explained by that parameter estimate.

General dominance is computed for parameter estimate $\beta_{dv_{iv}}$ ¹ by:

$$C_{\beta_{dv_{iv}}} = \sum_{i=1}^d \sum_{j=1}^{\binom{d-1}{i-1}} \frac{F_{\beta_{dv_{iv}}S_{i,j}} - F_{S_{i,j}}}{\binom{d-1}{i-1} d} \quad (1)$$

Where F is a fit metric chosen by the researcher (e.g., such as the McFadden pseudo- R^2 discussed below), $S_{i,j}$ is a distinct subset of the d parameter estimates that were estimated including/not constraining to 0 parameter estimate $\beta_{dv_{iv}}$, and $\binom{d-1}{i-1}$ is the number of distinct combinations of the size of the bottom number (e.g., $i-1$) out of number of elements of the size of the top number (e.g., $d-1$). Thus, $F_{\beta_{dv_{iv}}S_{i,j}}$ is the fit metric for the distinct subset of parameter estimates $S_{i,j}$ which also includes/estimates $\beta_{dv_{iv}}$ and $F_{S_{i,j}}$ is the fit metric for the same distinct subset of parameter estimates $S_{i,j}$ which excludes/constrains to 0 $\beta_{dv_{iv}}$. We recommend readers refer to Grömping (2007) and Azen and Budescu (2003) for more conceptual discussion of the general dominance statistic.

To illustrate Equation 1 more concretely, consider parameter estimate β_{YX_2} from the Model 2a in Figure 2. The general dominance statistic related to β_{YX_2} is computed as:

$$C_{\beta_{YX_2}} = \frac{F_{\beta_{YX_2}}}{\binom{3-1}{1-1} 3} + \frac{(F_{\beta_{YX_2}\beta_{X_2X_1}} - F_{\beta_{X_2X_1}}) + (F_{\beta_{YX_2}\beta_{YX_3}} - F_{\beta_{YX_3}})}{\binom{3-1}{2-1} 3} + \frac{(F_{\beta_{YX_2}\beta_{YX_3}\beta_{Y_2X_1}} - F_{\beta_{YX_3}\beta_{Y_2X_1}})}{\binom{3-1}{3-1} 3}$$

Using general dominance statistics, the researcher can determine whether one parameter estimate *generally dominates* another and is thus relatively more important. More broadly, general dominance is determined when the general dominance statistic for parameter estimate is bigger than that of another parameter estimate.

Prior DA research has recommended using (pseudo-) R^2 fit metrics as they are easy to interpret and meet several statistical fit metric criteria useful for decomposition in general dominance (Azen & Traxel, 2009). Of the metrics recommended in past work, the McFadden (1973) pseudo-

¹ Although we represent parameter estimates as regression slopes (e.g., dv_{iv}), implying a dependent variable (dv) to independent variable (iv) relationship, parameter estimates can include covariances as is outlined above and illustrated below.

R^2 is the only metric used to determine parameter estimate relative importance in the present study as it is the simplest. The McFadden pseudo- R^2 is derived as:

$$R_{McFadden}^2 = 1 - \frac{\ln L_{model}}{\ln L_{baseline}} \quad (2)$$

Where $\ln L_{baseline}$ is the log-likelihood of a baseline model where all the parameter estimates are constrained to 0—often this is a model containing only regression constants or intercepts. Other fit metrics can also be used for DA, but as Azen and Traxel (2009) show, any likelihood-based fit metric (e.g., Estrella pseudo- R^2 , Akaike information criterion) will result in the same general dominance statistics and relative importance determinations.

EXAMPLES DEMONSTRATING THE APPLICATION OF PARAMETER ESTIMATE RELATIVE IMPORTANCE

In this section, we walk through two examples applying the PERI determination methods described above. For this illustration, we estimate two models that are complementary exemplars of how PERI can be applied using DA. These two models are a linear, normally distributed path model and zero-inflated Poisson regression.

Data and Analysis Preparation

Data used in the present study were obtained from the General Social Survey (GSS) in years 2002, 2006, 2010, and 2014. These data were obtained from the GSS website located at <http://gss.norc.umd.edu/>. We recommend that interested readers refer to the GSS website for further study and variable information including the survey questionnaire wording and question stems, the sampling methodology, as well as the sample demographics by year. For all questions, any *Don't know*, *Not applicable*, or *No answer* responses were coded as missing and omitted from the analysis case-wise.

Five variables were chosen from the GSS data across all 4 survey years. The first variable was a work shift question (labeled *wrksched* in GSS). The work shift question asked respondents: *Which of the following best describes your usual work schedule?* Respondents reported on their work shift using the following categories: *Day shift*; *Afternoon shift*; *Night shift*; *Split shift*; *Irregular shift/on-call*; or *Rotating shifts*. The work shift question was recoded so that all day shift employees received a score of 1 (72% endorsed) and all other non-missing responses received a score of 0 (28%).

The second variable was a frequency of working at home question (labeled *wrkhome* in GSS). The work at home frequency question asked respondents: *How often do you work at home as part of your job?* Respondents reported on how often they work at home using the following scale: *Never* (coded 1; 60% endorsed), *A few times a year* (9%), *About once a month* (6%), *About once a week* (7%), *More than once a week* (12%), or *Worker works mainly at home* (coded 6; 6%). The work at home frequency question was recoded by subtracting 1 from each respondents' score so that *Never* responses corresponded to a value of 0.

The third variable was a work-to-family conflict question (labeled *wkvsfam* in GSS). The work-to-family conflict question asked respondents: *How often do the demands of your job interfere with your family life?* Respondents reported on how often work conflicted with family life on the following scale: *Often* (coded 1; 13% endorsed), *Sometimes* (30%), *Rarely* (31%), or *Never* (coded 4; 26%). The work-to-family conflict question was reverse-coded so that higher scores indicated more frequent conflict with family life.

The fourth variable was a schedule change capability question (labeled *chngtme* in GSS). The schedule change capability question asked respondents: *How often are you allowed to change your starting and quitting times on a daily basis?* Respondents reported on how often they could change their work schedule on the following scale: *Often* (coded 1; 33% endorsed), *Sometimes* (20%), *Rarely* (15%), or *Never* (coded 4; 31%). The schedule change capability question was reverse-coded so that higher scores indicated more frequent ability to change schedule.

The fifth variable was a permanent employee status question (labeled *wrktype* in GSS). The permanent employee status question asked respondents: *How would you describe your work arrangement in your main job?* Respondents reported on their work arrangement using the following categories: *Independent contractor/consultant/freelance worker*; *On-call, work only when called to work*; *Paid by temporary agency*; *Work for contractor who provides workers/services under contract*; or *Regular, permanent employee*. The permanent employee status question was also coded so that all regular, permanent employees received a score of 1 (80% endorsed) and all other non-missing responses received a score of 0 (20%).

Path Model Example

The path model estimated is represented in Figure 3; note that each path is numbered in Figure 3 and Table 1 for easier comparison. The analysis began with estimating parameters, both unstandardized and standardized; all path model slope and covariance results are reported in the β and β_{Std} columns of Table 1. The path model estimated fit to the data well and obtained model selection indexes that were within most established standards (CFI = .97; TLI = .91; RMSEA = .05; $p_{RMSEA} = .30$).

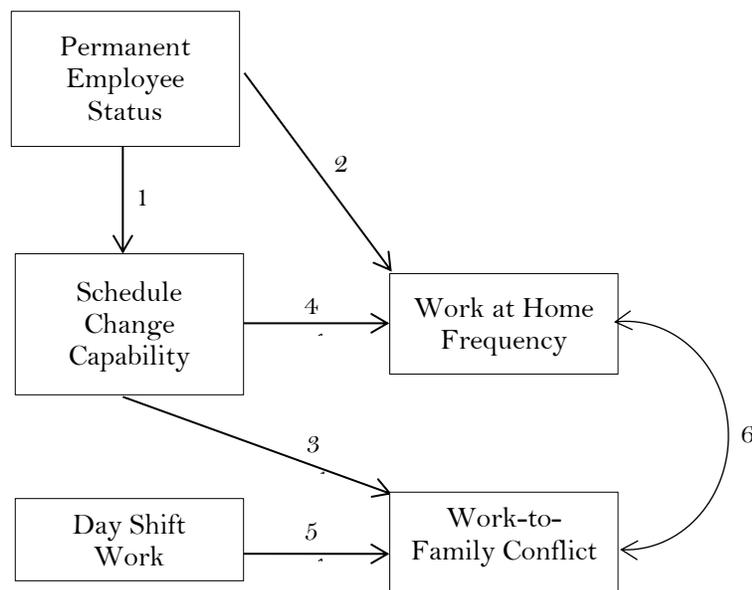


Figure 3. Diagram of Path Model from Data Analytic Example

Table 1. Path Model Results

Parameter Estimates	β	β_{Std}	C_{β} (rank)	Independent Variables	C_x (rank)
Schedule Change Capability					
1. Permanent Employee Status	-0.6662	-.2168	.0039 (3)	Permanent Employee Status (1 and 2)	.0098 (1)
Work-to-Family Conflict					
3. Schedule Change Capability	0.0479	.0596	.0002 (6)	Schedule Change Capability (3 and 4)	.0083 (2)
5. Day Shift Work Work at Home	-0.2259	-.1017	.0008 (5)		
4. Schedule Change Capability	0.3929	.2833	.0083 (1)	Day Shift Work (5)	.0008 (3)
2. Permanent Employee Status	-0.9576	-.2246	.0057 (2)		
Correlation					
6. Work at Home-Work-to-Family Conflict ¹	0.3059	.1967	.0031 (4)		
McFadden's R ²			.0221		.0221

N = 5,834. All parameter estimates are statistically significant $p < .05$. β_{Std} = Standardized Coefficient. C_{β} = Parameter estimate-based general dominance statistic. C_x = Independent variable-based general dominance statistic. ¹Correlation controlled for in independent variable-based DA and obtained a value of .0031. Note: path numbers in Figure 3 noted for PERI analysis and paths combined for the independent-variable analysis noted in parentheses.

The path model parameter estimates obtained indicated that permanent employees had less schedule change capability (path 1 in Figure 3) and worked at home less frequently (path 2) than non-permanent employees. Permanent employees thus were more likely to work a structured set of hours and to work primarily at the work site than their non-permanent counterparts. In addition, schedule change capability also resulted in more work-to-family conflict (path 3) and to working at home more frequently (path 4). The flexibility in work schedule then led to working more at home as well as increased conflict with family life possibly as a result (discussed below). Finally, day shift work was related to less work-to-family conflict (path 5). In combination with the regression estimates, the covariance between work-to-family conflict with the frequency of working at home was positive (covariance path 6); individuals who worked at home more were more likely to report that their work life conflicted with their family life than those who worked at home less.

PERI AND STANDARDIZED PARAMETER ESTIMATES COMPARISON

With the model results above, we now move to consider which of the paths in Figure 3 are most important for understanding the conceptual relationships revealed by this model; that is, PERI. To this end, consider first the standardized slope coefficients in Table 1. The standardized results suggest that the relationship between schedule change capability and work at home frequency was the most important relationship in the model as it was largest. Thus, in terms of understanding workers' home life and work shift behaviors, the most impactful result has to do

with knowing whether a worker can change their shift and the impact of that capability for working at home.

The general dominance statistics in Table 1 associated with each parameter estimate agree with the rank ordering of the standardized slope coefficients in terms of their relative magnitude. Specifically, the schedule change capability predicting work at home frequency was also determined to be the most important parameter estimate in the model with the general dominance statistics. Whereas the rank order of the parameter estimates was the same across standardized parameter estimates and general dominance statistics, there was additional information offered by the general dominance statistic which cannot be obtained in a straightforward way from standardized parameter estimates. Specifically, the general dominance statistics provide a realistic estimate regarding the extent to which each parameter estimate was associated with explanatory usefulness in terms of a component of model fit. As applied to the most important schedule change capability predicting work at home frequency relationship, the general dominance statistic suggests that this relationship explained 38% of the model fit (i.e., $.0083/.0221$). Thus, a great deal of the explanatory usefulness in the model derived from this relationship. By contrast, the standardized slope coefficient showed that a one standard deviation increase in schedule change capability is associated with an approximately one-quarter standard deviation change in work at home frequency; this is (perhaps) equally useful but qualitatively different information.

In addition, consider the two strongest parameter estimates schedule change capability and permanent employee status predicting work at home frequency. Their standardized slope coefficients suggest that schedule change capability was 26% stronger (e.g., $\text{abs}(.2833/-.2246)$) in terms of prediction. Thus, the rate of change in predicting working at home for schedule change capability increased at a rate that is 26% faster than that of permanent employee status. By comparison however, the general dominance statistics suggest that schedule change capability was 46% stronger (i.e., $.0083/.0057$) in terms of explaining model fit across the entire range of schedule change capability and permanent employee status. Thus, considering the amount of variability in schedule change capability, its impact on work at home frequency was 46% greater than the impact of permanent employee given its variability. The comparisons do not agree as they are comparing different quantities, ratios of predicted values in the case of standardized parameter estimates versus ratios of model fit components in the case of general dominance statistics.

The distinction between standardized parameter estimates and general dominance statistics is especially stark when considering the second and third ranked parameter estimates. The standardized slope coefficients suggest that permanent employee status predicting work at home frequency is 4% (i.e., $\text{abs}(-.2246/-.2168)$) stronger than permanent employee status predicting schedule change capability. However, the general dominance statistics showed a 46% difference (i.e., $.0057/.0039$) as, again, they are comparing different quantities. As applied to these two parameter estimates, the standardized slopes are also conceptually less comparable as they describe prediction of two different dependent variables. By comparison, the general dominance statistics describe components of the model fit and, conceptually, are far easier to compare as they are derived from the same metric.

One potentially surprising result from this analysis is that the covariance between work-to-family conflict and work at home frequency resulted in the fourth ranked parameter estimate, generally dominating all the slope parameters involving work-to-family conflict. The importance of the covariance implies one of two things about the model. First, work at home frequency may predict work-to-family conflict better than day shift work or schedule change capability. Conversely, the opposite may be true, work-to-family may predict work at home frequency least strongly compared to permanent employee status and schedule change capability - the direction of the effect is unknown. Second, there may be a more complex network of associations tying together

work-to-family conflict and work at home frequency that is not explained by the path model in the current study. The nature of the relationship between work-to-family conflict and work at home frequency is something that can be left for future research; the current study shows it could be important and useful to explain.

PERI and IVRI comparison

Although the PERI results are the focus of the present work, for the purposes of demonstration, we also compare the PERI results to those from an IVRI approach. To recapitulate, IVRI groups parameter estimates by independent variable to determine the relative importance of each independent variable in explaining model fit. The results related to independent variables, grouped by path number, are reported in the C_x column in Table 1. As compared to the PERI results, the IVRI determinations point to permanent employee status as most important in the context of the entire model. This is an interesting finding as the PERI results flagged schedule change capability as being associated with the single strongest parameter estimate in the model (i.e., the schedule change predicting work at home frequency parameter estimate). Thus, despite the fact that schedule change capability was associated with the most important single parameter estimate, the combination of parameter estimates associated with permanent employee status (i.e., the second and third ranked parameter estimates) was determined more important with IVRI.

The comparison between PERI and IVRI shows the relative strengths of both for understanding model fit. PERI offers a detailed depiction of each parameter estimate which can, and often does, show differences in terms of prediction across independent and dependent variable pairs. By contrast, IVRI groups parameter estimates to obtain determinations of importance for a single independent variable. Such IVRI determinations provide results at a higher-level that may be simpler to interpret, but its values will depend on how many parameter estimates are associated with each independent variable as well as which dependent variables are predicted by that independent variable.

An important note about IVRI results is that IVRI results are also similar to, but not identical with, the sums of PERI results. For example, the value associated with permanent employee status IVRI results (i.e., combining paths 1 and 2 in Figure 3) results is .0098. When summing the PERI results we obtained .0096 (i.e., .0039 + .0057). As such, IVRI results can be anticipated from PERI results when summed by independent variable. A final important point of note is that IVRI cannot, in a conceptually straightforward way, handle a covariance where there is no independent-dependent variable distinction. In IVRI, the covariance in (i.e., path 6) was treated as a factor which was adjusted for (by including in the full-factorial of models) but not directly dominance analyzed.

To conclude the path analysis example, the results of the path model in Table 1 point to the work at home frequency dependent variable as a component of the model that is most important and explainable. That is, the model in Figure 3 provides the most information about work at home frequency. Consequently, the example outlined in the next section expanded on the prediction of work at home frequency using a different model to provide more nuanced sets of conclusions. In the section to come the work at home frequency question was modeled using a zero-inflated Poisson regression.

Zero-inflated Poisson Regression Example

This second analysis looked more closely at the work at home frequency dependent variable to better understand reasons for different work at home frequencies. As is reported above, the work at home frequency question was highly skewed with 60% of respondents reporting that they never work at home. To account for the high number of “never” responses, the work at home

frequency variable was fit to a Poisson distribution where there are excessive 0's—that is, “never” work at home responses. The “never”-inflation equation was a logistic model being predicted by schedule change capability, permanent employee status, work-to-family conflict, and day shift work. The frequency of working at home conditional on the “never”-inflation model was a Poisson model predicted by permanent employee status and work-to-family conflict.

Table 2. Zero-inflated Poisson Regression Results

Parameter Estimates	β	e^{β}	C_{β} (rank)	Independent Variables	C_x (rank)
Work at Home Frequency					
Permanent Employee Status	-0.3765	0.6863	.0131 (3)	Schedule Change Capability	.0434 (1)
Work-to-Family Conflict	0.0576	1.0593	.0017 (6)		
Never Work at Home-Inflation				Permanent Employee Status	.0214 (2)
Schedule Change Capability	-0.6287	0.5333	.0432 (1)		
Permanent Employee Status	0.6600	1.9348	.0085 (4)	Work-to-Family Conflict	.0161 (3)
Work-to-Family Conflict	-0.4872	0.6143	.0144 (2)		
Day Shift Work	-0.4947	0.6098	.0025 (5)	Day Shift Work	.0024 (4)
McFadden's R^2			.0833		.0833

$N = 5,834$. All parameter estimates are statistically significant $p < .05$. e^{β} = Exponentiated coefficient. C_{β} = Parameter estimate-based general dominance statistic. C_x = Independent variable-based general dominance statistic.

The parameter estimate values obtained by the zero-inflated Poisson are reported in the β column of Table 2. The results of the zero-inflated Poisson regression were similar to those obtained in the path model results as the frequency of working at home increased with more work-to-family conflict but decreased for permanent employees. Thus, respondents who reported work-to-family conflict worked at home more frequently and permanent employees worked at home less frequently.

An interesting and useful component of the zero-inflated Poisson was the “never”-inflation equation predicting the likelihood of obtaining a “never” response independent of their frequency of working at home. Work-to-family conflict as well as permanent employee status also predicted the inflation equation independent of the frequency equation and in the same direction as their parameter estimate in the work at home frequency equation. That is, work to family conflict both decreased the likelihood of reporting “never” working from home as well as increased the frequency of working from home conditional on the “never”-inflation results. Similarly, permanent employees were both more likely to report “never” working from home as well as reported working from home less frequently conditional on the “never”-inflation results. In addition, schedule change capability and day shift work obtained negative results on the “never”-inflation equation which decreased the likelihood of responding “never” to working at home.

PERI and exponentiated parameter estimates comparison

The results from the zero-inflated Poisson regression were not straightforward to compare across the work at home (i.e., Poisson) and “never”-inflation (i.e., logit) equations as they differed in the way they were scaled. The work at home frequency equation is based on a Poisson distribution with a natural logarithm link function and had a lower bound of 0 in terms of how they translate into predicted frequencies. The inflation equation was based on a binomial distribution with a

logistic link function which had a bound between 0 and 1 in terms of how they translate into predicted probabilities. In addition, zero-inflated Poisson regressions do not have simple standardization methods to make parameter estimates more comparable in the same way as path models. The parameter estimates which permit the most simple comparison for a zero-inflated Poisson regression are exponentiated parameter estimates (e.g., Greenland & Maldonado, 1994). Exponentiated parameter estimates for the Poisson produced incidence rate ratios (i.e., percent increase in the incidence rate of the dependent variable for every one-point increase in the independent variable). Exponentiated parameter estimates for the zero-inflation equation produced odds ratios (i.e., the increase in the odds of having a zero in the dependent variable for every one-point increase in the independent variable). Both types of exponentiated parameter estimates are reported in the e^{β} column of Table 2. As can be seen in Table 2, comparing exponentiated parameter estimates was difficult across positive and negative effects as such effects ranged from zero to one in the negative range and one to infinity in the positive range. This is a fundamental drawback of the use of exponentiated parameter estimates to infer importance as the direction of the effect can result in differently scaled statistics. Irrespective, when ranked by exponentiated parameter estimates, the strongest effect appeared to be associated with permanent employee status predicting “never” inflation which resulted in an increase in the odds of responding “never” by 93% for permanent employees relative to non-permanent employees.

The results from the PERI-based DA reported in the C_{β} column of Table 2 show that the exponentiated parameter estimates rank ordering does not align well with that obtained from the general dominance statistics. The most important parameter estimate in the model according to the general dominance statistics was the schedule change capability predicting “never” inflation. In fact, this parameter estimate alone explained a little over half of the model fit (52%; .0432/.0833) in disagreement with the exponentiated parameter estimates where it appeared to be second ranked. By contrast, the permanent employee status predicting “never” inflation parameter estimate, which appeared most important according to the exponentiated parameter estimates, ranked fourth among the parameter estimates explaining 10% of the model fit (.0085/.0833). As with the standardized parameter estimates, exponentiated parameter estimates described the rates of change of predicted values compared to general dominance statistics which described components of model fit. In the case of the zero-inflated Poisson, the dominance statistics were clearly advantageous for comparing parameter estimates given their shared basis on a model fit metric and consequent easy comparability.

Before transitioning to discussing IVRI, it is useful to compare permanent employee status predicting work at home frequency versus permanent employee status predicting “never”-inflation. Permanent employee status was clearly more informative about generating “never”-inflation than it was about predicting the frequency of working at home. Additionally, work-to-family conflict can be compared across equations in a similar way with a similar result, that work-to-family conflict is associated with “never”-inflation more than work at home frequency. Taken together, these two comparisons show a distinct advantage of PERI statistics: being able to compare the same independent variable across very different predictive equations, something importance methods to this point have not been able to do in a straightforward way. Finally, the PERI results for the zero-inflated Poisson also show that “never” inflation is far more predictable than work at home frequency. That is, this model speaks more to why people cannot work at home overall than the frequency with which they work at home when they can.

PERI and IVRI comparison

The results of the IVRI determination are reported in the C_x column of Table 2. The independent-variable-based results reveal information similar to that obtained from the PERI results in that schedule change capability explains approximately half of the model fit. Thus,

despite being associated with only a single parameter estimate, schedule change capability remained the most important independent variable, generally dominating work to family conflict and permanent employee status.

Permanent employee status also dominated work to family conflict despite work to family conflict being associated with the second most important parameter. Permanent employee status's two parameters, when combined, overshadowed work to family conflict's two parameters—most likely owing to the low explanatory usefulness of the work to family conflict predicting work at home frequency parameter. Importantly, and as noted above, most IVRI results can be anticipated by knowing PERI results; this was true of the zero-inflated Poisson regression results as it was of the path model.

Practical Considerations in Applying this DA to PERI Determination

To this point, the paper has centered on the fundamental conceptual and statistical issues regarding PERI for multivariate models. Here, we discuss several assumptions and content decisions necessary to make this approach manageable in scale. Firstly, the recommended method — DA— produces several statistics, one of which is the general dominance statistic. Two other statistics are available: conditional and complete dominance (see Budescu & Azen, 2004). Conditional and complete dominance statistics are more stringent and thus stronger relative importance determinations than general dominance statistics. In addition, conditional and complete dominance are more specific in the information about how and where parameter estimates and independent variables explain model fit. General dominance was chosen above as it involves the computation of a single statistic and can almost always be used to determine relative importance between two parameter estimates. Conditional dominance statistics are computed at different numbers of parameter estimates in the model, and complete dominance statistics compare each parameter estimate to each other parameter estimate on a pairwise basis. Both additional dominance statistics result in more comparisons and require more interpretation by the researcher.² Discussing general dominance only allowed for a condensed discussion of the proposed method but it should not be viewed as the only dominance statistic to determine PERI.

In addition, the PERI determinations here did not sum to result in the IVRI determinations. The reason for the discrepancy is that more models were involved in the PERI computations than IVRI computation. Any independent variable with multiple parameter estimates in the model would have had estimated models and overall fit estimates which include subsets with combinations of the otherwise grouped parameter estimates being omitted. For example, there were sixty-three models involved in the PERI determination for the path analysis yet only seven for the IVRI determination. The number of models, and general dominance statistics computed from those models, will only be identical when all the models included in the PERI approach are included in the IVRI approach.

We recommend researchers interested in determining IVRI for multivariate models use the method applied in the present work. Specifically, grouping parameter estimates by independent variable and using a maximum likelihood model with parameter estimate constraints that force parameters in the model to 0. The dominance statistics can then be computed using the model fit metrics computed from the parameter estimate groups reflecting each independent variable.

DISCUSSION

² These statistics for the current examples are available by request from the first author.

The goal of this study was to extend methods for determining relative importance to a more general set of multivariate models using the PERI approach. PERI is adapted from IVRI but focuses on contributions to model fit by parameter estimates. To recapitulate, the characteristics of PERI are: a) relative importance statistics are based on a single parameter estimate, b) relative importance statistics are aggregated across nested models within the same overarching statistical model, and c) relative importance statistics are based on comparisons of an overall model fit metric.

As we showed in the present work, PERI and IVRI determinations serve different analytic purposes. IVRI is informative in that it reflects total effect of an independent variable on the model, which is more suitable if the focus of the analysis is practical or applied in nature. Such situations might arise when evaluating practical applications such as personnel selection or evaluating behavioral therapy interventions. By contrast, PERI is focused more on understanding the model. PERI helps to understand the model as it permits an in-depth comparison of each path on a shared and easily comparable metric. Ultimately, as was shown in the examples above, independent variable IVRI statistics are essentially an aggregate of PERI statistics. As a consequence, a key difference between PERI and IVRI is conceptual — PERI statistics were developed in this work to simultaneously broaden the scope of what qualifies as an importance statistic as well as focus on the contribution each parameter estimate makes to model fit.

Beyond the conceptual difference, in the building of the PERI approach, we have pointed out some key limitations of IVRI for multivariate models. As is illustrated in the introduction, by separating importance determination by parameter estimate PERI avoids nesting of independent variables and dependent variables and is not influenced by unequal number of parameters aggregated with each independent variable such as occurred in the path model example. Moreover, PERI serves as a highly useful method for comparing parameters in models with multiple different functional forms and probability distributions (like the zero-inflated Poisson example, when the dependent variable has excessive zeros).

This study recommended a methodology for determining PERI using DA statistics. PERI-based DA computes dominance statistics by means of imposing estimation constraints to omit a parameter or estimate it from the data. In the present work, we outlined a path model example that applied PERI statistics to paths in the path model and compared PERI statistics to standardized regression slope statistics for model interpretation. As was noted, PERI statistics are advantageous for interpretation over standardized regression slopes as PERI statistics take into account both the slope size as well as variability in the independent and dependent variables all at once (Grömping, 2007) whereas standardized slopes focus on predicted values. The second example of zero-inflated Poisson regression showed the advantages of PERI-based DA statistics over exponentiated parameter estimates for comparing different parameters across predictive equations with different probability distributions. In both examples, IVRI was also computed and compared to PERI-based statistics. These comparisons showed that IVRI are tantamount to sums of the PERI-based versions and, furthermore, how different numbers of parameter estimates associated with an independent variable can complicate comparisons of independent variable importance as more parameters aggregated for an independent variable tend to produce a larger IVRI statistic.

IMPLICATIONS AND FUTURE RESEARCH

In our view, the primary contribution of this work is in demonstrating the utility of PERI as the basis for determining importance in many types of linear or generalized linear statistical models. Beyond the two examples described here, PERI could also be applied to various other complex multivariate models. One particularly useful application would be to multilevel or mixed effects

regressions. At current, IVRI methods have been developed for multilevel models with two levels of aggregation (Luo & Azen, 2013). This method separates the levels of aggregation when computing importance statistics and thus does not permit comparison of parameters across levels. It is feasible that a PERI approach could be developed to apply to such multilevel models and, furthermore, could be used to compare the effects of an independent variable which exists at two levels of aggregation simultaneously (i.e., collective efficacy vs self-efficacy). Future research describing the stages of such a cross-level PERI-based DA and outlining best practices for such an analysis would be useful for understanding individual vs. collective constructs as well as within-person vs. between-person effects.

An important issue to address in future research using multilevel, as well as structural equation, models is how to accommodate models with random effects and latent variables in a general way. Latent variables and random effects are estimated from the data and cannot be directly accommodated in model fit metrics such as the McFadden pseudo- R^2 used in the present study. This is because fit metrics such as the McFadden pseudo- R^2 are fit to observed values only. The issue to be addressed is related to the complication that latent variables and random effects are distributions and do not have real values against which to benchmark prediction, as do observed data. This point is clearest when using Bayesian estimators, where all estimates are random effects, and they do not produce single predicted values (see Little, 2006). As is noted above, Luo and Azen (2013) have developed methods to determine importance for special cases of multilevel models with two levels of aggregation. However, there has been little work to extend Luo and Azen's approach to more general cases as well as to latent variable models. Future research adapting fit metrics to appropriately assess overall model fit in the presence of latent variables and random effects as well as research to accommodate parameter estimates obtained from latent variables and random effects will be crucial for effectively extending PERI to such models. At current, we do not recommend the application of PERI-based DA as outlined here to models with latent variables or random effects. It is worth noting that research has begun extending relative importance methods into Bayesian estimators (e.g., Shou & Smithson, 2015) using model averaging. Such an approach could apply to PERI estimation and is an interesting possible extension for future research.

An additional, and ongoing, problem for ensemble methods such as DA is the possibility for large numbers of parameter estimates in a statistical model. Multivariate models can accommodate a great number of parameter estimates which can lead to practical complications and computational limits when the number of models to be estimated begins to grow exceedingly large. This study walked through two examples with six parameter estimates each example which produced a total of $2^6 - 1$ or sixty-three different models to estimate and aggregate per example. Given the base of two, the number of models to estimate approximately doubles with each parameter estimate included. For example, a model with twelve parameter estimates requires $2^{12} - 1$ or four thousand ninety-five different models to incorporate into DA statistics - hence, having more parameter estimates in a model means that the DA method becomes less computationally feasible as the number of estimates grows near 20 for many modern computers. Relative weights analysis (Johnson, 2000), a similar but more computationally efficient method than DA, could be used for IVRI applications in these cases. Although relative weights analysis has been applied to many of the same analyses as has DA, the orthogonalization methods used by relative weights analysis cannot be applied to the parameter estimation problem — the focus of the present work. Obtaining relative weights requires an analysis which can be estimated or well-approximated by matrix manipulations like least squares estimation. The multivariate models outlined in the present work require maximum likelihood estimation. Thus, applying relative weights to PERI may not be possible. One potentially useful way to estimate PERI statistics using DA could be to obtain a subset of the full-factorial of combinations of parameter estimate constraints which results in a sufficiently close approximation to the full-factorial of parameter estimate constraints. For example, it might be feasible to obtain a well-chosen combination of models that represents

a fraction of the full-factorial of models yet permits the computation of accurate dominance statistics. We expect that the application of methods similar to those in the fractional factorial (e.g., Collins, Dziak, & Li, 2009) design literature might be fruitful directions for this research.

Finally, the current work did not address the possibility for determining the importance of indirect effects in a path model. Because indirect effects are based on functions of individual parameter estimates, a simple way to understand the importance of any given indirect effect is simply to sum the impact of the component parameter estimates. Indirect effects, as combinations of individual parameter estimates, otherwise would have to be grouped like the IVRI approach. However, doing so would eliminate the possibility of independently estimating the relative importance of all component parameters and does not allow for multiple indirect effects on any of the same component parameters. Future research formulating a better answer to how to estimate the importance of indirect effects would be beneficial. Provisionally, the recommended approach is, again, to sum the individual PERI statistics by indirect effect pathway.

CONCLUSION

This paper has sought to broaden the conceptualization of importance in a statistical model by developing the concept of PERI. PERI extends beyond the predominant IVRI methods and, as we have shown, IVRI is tantamount to a subset of PERI results. We have also shown that PERI extends beyond standardized coefficients in that it offers a more comparable metric across dependent variables and focuses on the entire continuum of the independent variable as opposed to single standard deviations at a time. As such, PERI analysis offers researchers and decision-makers a new way to compare parameter estimate effects across predictive equations and dependent variables which enhance understanding of statistical model results. We believe that PERI is a concept which can be extended to many novel relative importance situations across a wide variety of models and can allow easier comparisons of parameters across parameter estimates with widely different scales and functional forms.

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